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Periodic free convection from a vertical plate in a saturated porous medium, non-equilibrium model

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Abstract

The problem of the free convection from a vertical heated plate in a porous medium is investigated numerically in the present paper. The effect of the sinusoidal plate temperature oscillation on the free convection from the plate is studied using the non-equilibrium model, i.e., porous solid matrix and saturated fluid are not necessary to be at same temperature locally. Non-dimensionalization of the two-dimensional transient laminar boundary layer equations results in three parameters: (1) H, heat transfer coefficient parameter, (2) K_r, thermal conductivity ratio parameter, and (3) λ, thermal diffusivity ratio. Two additional parameters arise from the plate temperature oscillation condition which are the non-dimensional amplitude (ε) and frequency (Ω). The fully implicit finite difference method is used to solve the system of equations. The numerical results are presented for $0 \leqslant H \leqslant 10$, $0 \leqslant K_r \leqslant 10$, $0.001 \leqslant \lambda \leqslant 10$ with the plate temperature oscillation parameters $0 \leqslant \Omega \leqslant 10$ and $0 \leqslant \varepsilon \leqslant 0.5$. The results show that the thermal conductivity ratio parameter is the most important parameter. It is found also that increasing the amplitude and the frequency of the oscillating surface temperature will decrease the free convection heat transfer from the plate for any values of the other parameters.

1. Introduction

The fundamental free convection along vertical/inclined heated/cooled plate has been studied extensively for both pure fluids and porous media by various authors. The laminar boundary layer approximation is usually used in the analysis due to its wide of engineering applications. Representative studies in this area may be found in the recent books by Nield and Bejan [1], Vafai [2], Pop and Ingham [3] and Bejan and Kraus [4]. Most authors considered the isothermal or stream wise temperature variation of the plate. But in the industrial applications, quite often the free convection is a time dependent or periodic process. The practical free convection problem with the periodic oscillation of the surface temperature has been addressed by Das et al. [5]. They have used Laplace transform technique to solve the simplified equations, and the results show that the transient velocity profile and the penetration distance decreases with increasing the frequency of the plate temperature oscillation. Recently Saeid [6] has considered

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Nomenclature

$A(Nu)$	amplitude of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$
C_p	specific heat (J kg^{-1} K^{-1})
g	gravitational acceleration (m s^{-2})
h	heat transfer coefficient between the solid and fluid phases (W m^{-3} K^{-1})
H	heat transfer coefficient parameter $H = hL^2/(\varphi k_f Ra)$
k	thermal conductivity (W m^{-1} K^{-1})
K	permeability (m^2)
K_r	thermal conductivity ratio parameter $K_r = \varphi k_f/\{(1-\varphi)k_s\}$
L	plate height (m)
$\overline{Nu}$	average Nusselt number
$\overline{\overline{Nu}}_t$	temporal cyclic averaged value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$
Ra	Darcy–Rayleigh number
t	time (s)
T	temperature (K)
u, v	velocity components along and normal to the plate respectively (ms^{-1})
U, V	non-dimensional velocity components
x, y	Cartesian coordinates along and normal to the plate respectively (m)
X, Y	non-dimensional Cartesian coordinates

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
β	coefficient of volume expansion (K^{-1})
ε	non-dimensional amplitude
ν	kinematic viscosity (m^2 s^{-1})
ρ	density (kg m^{-3})
θ	non-dimensional temperature
λ	thermal diffusivity ratio $\lambda = \alpha_f/\alpha_s$
τ	non-dimensional time
φ	porosity
ω	frequency (s^{-1})
Ω	non-dimensional frequency

Subscripts

f	fluid
s	solid
t	total (fluid + solid)
w	wall
∞	ambient

the same problem with relatively higher Grashof number is considered ($10^4 < Gr < 10^9$) where the laminar boundary layer theory is applicable to study the effect of periodic plate temperature oscillation on the free convection from vertical plate to pure viscous fluids (air and water). The results show that increasing the amplitude and the frequency of the oscillating surface temperature will decrease the free convection heat transfer from the plate to both air and water. For the free convection in porous media, various authors [7,8] considered the porous matrix is in thermal equilibrium with the fluid, i.e., the temperature of solid and fluid are assumed to be the same within the representative control volume. Thermal equilibrium is not valid when the heat is released in the solid or in the fluid phase. For instance, in the case of combustion in a porous matrix, the heat is released in the combustible gases. The difference between temperatures of the solid and gases can be quite substantial in the flame region ([9]). In adsorption/desorption process or in a catalyst converter, the heat is released mainly in the solid matrix. Also, in high speed and/or high Darcy flow applications, it is important to account for thermal non-equilibrium effects ([10]). Furthermore, when the length scale of the representative control volume is the same order of the length of the system, then the thermal equilibrium model prediction may be unacceptable ([11]). It is expected that, when there is a significant difference between advection and conduction mechanisms in transferring heat, the deviation between solid and fluid phase temperatures increases.

Schumann [12], suggested a simple two-equation model to account for non-equilibrium condition for incompressible forced flow in a porous medium. Vafai and Sozen [10], extended the Schumann model to account for compressible flow taking into account of Forchheimer term and conduction effects in the gas and solid phases. Amiri and Vafai [13] presented a detail analysis for forced flow through channel filled with saturated medium. They considered the effects of porosity variation, boundary effects, inertial effects and non-equilibrium condition. Their results indicate that the Darcy and particle Reynolds parameters are most influential parameters in determining the validity of the local thermal equilibrium. Recently the non-equilibrium model has been used in the analysis of different convection heat transfer problems in porous media by various authors [14–25]. In the non-equilibrium modeling, it is required to know the volumetric heat transfer coefficient between solid and fluid phases. In the literature there are some attempts to measure volumetric heat transfer coefficient indirectly under forced convection conditions ([26,27]). These methods are based on the thermal response of the system to thermal pulses or by measuring the transient response of the system and use of a non-linear least square method to match the prediction of the governing equations and experimental data (inverse problem). Wu and Hwang [28], published experimental data for heat transfer between air and solid particles in a porous bed. The correlated results showed that the local Nusselt parameter is a function of the Reynolds parameter and the bed porosity. The local Nusselt parameter (based on the particle diameter and thermal conductivity of the fluid) can be as low as 1.0 for highly porous layer and for *Re* on the order of 1000. For bed of porosity of 0.37–0.38 and for *Re* of 40–1000, the local Nusselt is between 10 and 100. In fact, the results depend on the accuracy of the model assumptions and accuracy of the input parameters, such as thermophysical properties of the solid matrix, effective thermal conductivity, boundary conditions and effect of radiation, etc. No experimental or theoretical analysis could be identified in the literature about volumetric heat transfer coefficient under natural convection conditions. From the basics of heat transfer, it is expected that the volumetric heat transfer coefficient for natural convection may be quite low compared with forced convection, unless the Rayleigh parameter is very high ($Ra > Re^2$). Mohamad [29] has analyzed the free convection from a vertical heated plate in a saturated porous medium using Darcy model and the non-equilibrium model. The predicted results are interesting and indicate that the equilibrium model is difficult to justify when the solid matrix thermal conductivity is higher than the thermal conductivity of fluid phase. In the present work, the Darcy model and the non-equilibrium model are used also to study the free convection from a vertical plate immersed in a porous media driven by sinusoidal plate temperature oscillation.

2. Governing equations

The conservation equations for mass, momentum and energy in two-dimensional, laminar boundary layer flow along vertical plate immersed in a porous media, using the non-equilibrium model are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{g\beta K}{\nu}\frac{\partial T_{\mathrm{f}}}{\partial y} \tag{2}$$

$$\varphi(\rho c_p)_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial t} + (\rho c_p)_{\mathrm{f}}\left(u\frac{\partial T_{\mathrm{f}}}{\partial x} + v\frac{\partial T_{\mathrm{f}}}{\partial y}\right) = \varphi k_{\mathrm{f}}\frac{\partial^2 T_{\mathrm{f}}}{\partial y^2} + h(T_{\mathrm{s}} - T_{\mathrm{f}}) \tag{3}$$

$$(1-\varphi)(\rho c_p)_{\mathrm{s}}\frac{\partial T_{\mathrm{s}}}{\partial t} = (1-\varphi)k_{\mathrm{s}}\frac{\partial^2 T_{\mathrm{s}}}{\partial y^2} + h(T_{\mathrm{f}} - T_{\mathrm{s}}) \tag{4}$$

where the vertical wall is considered to be along x-axis and y-axis is normal to it. In accordance with the present problem, the initial and boundary conditions are

$$u(x,y,0) = v(x,y,0) = 0; \quad T_{\mathrm{f}}(x,y,0) = T_{\mathrm{s}}(x,y,0) = T_\infty \tag{5a}$$

$$u(0,y,t) = v(0,y,t) = 0; \quad T_{\mathrm{f}}(0,y,t) = T_{\mathrm{s}}(0,y,t) = T_\infty \tag{5b}$$

$$v(x,0,t) = 0; \quad T_{\mathrm{f}}(x,0,t) = T_{\mathrm{s}}(x,0,t) = T_{\mathrm{w}}(t) \tag{5c}$$

$$u(x,\infty,t) = 0; \quad T_{\mathrm{f}}(x,\infty,t) = T_{\mathrm{s}}(x,\infty,t) = T_\infty \tag{5d}$$

The wall temperature condition is assumed to oscillate periodically over an average value $\overline{T}_{\mathrm{w}}$ with small amplitude ε and frequency ω so that the boundary layer theory is still valid. Therefore the following wall temperature condition is used:

$$T_{\mathrm{w}}(t) = \overline{T}_{\mathrm{w}} + \varepsilon(\overline{T}_{\mathrm{w}} - T_\infty)\sin\omega t \tag{6}$$

It is assumed that the fluid and solid phases have the same temperature at the vertical plate, i.e., equilibrium condition is imposed on the non-permeable plate. In fact, this boundary condition is not valid, due to the channeling effect. In the present work, the issue of non-equilibrium boundary condition is not considered due to lack of information.

In order to simplify the problem and to generalize the results for the laminar boundary layer flow along a vertical plate, the above equations are written in a non-dimensional form by employing the following boundary layer dimensionless variables:

$$\tau = \frac{t\alpha_{\mathrm{f}}}{L^2}Ra; \quad X = \frac{x}{L}; \quad Y = \frac{y}{L}\sqrt{Ra};$$
$$\Omega = \frac{\omega L^2}{\alpha_{\mathrm{f}}Ra}; \quad U = \frac{uL}{\varphi\alpha_{\mathrm{f}}Ra}; \quad V = \frac{vL}{\varphi\alpha_{\mathrm{f}}\sqrt{Ra}};$$
$$\theta_{\mathrm{f}} = \frac{T_{\mathrm{f}} - T_\infty}{\Delta T}; \quad \theta_{\mathrm{s}} = \frac{T_{\mathrm{s}} - T_\infty}{\Delta T} \tag{7}$$

where $\Delta T = \overline{T}_{\mathrm{w}} - T_\infty$ and Ra is the Darcy–Rayleigh number defined as $Ra = \frac{g\beta\Delta TLK}{\varphi\nu\alpha_{\mathrm{f}}}$. The non-dimensional forms of the governing Eqs. (1)–(4) are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{8}$$

$$U = \theta_{\mathrm{f}} \tag{9}$$

$$\frac{\partial\theta_{\mathrm{f}}}{\partial\tau} + U\frac{\partial\theta_{\mathrm{f}}}{\partial X} + V\frac{\partial\theta_{\mathrm{f}}}{\partial Y} = \frac{\partial^2\theta_{\mathrm{f}}}{\partial Y^2} + H(\theta_{\mathrm{s}} - \theta_{\mathrm{f}}) \tag{10}$$

$$\lambda\frac{\partial\theta_{\mathrm{s}}}{\partial\tau} = \frac{\partial^2\theta_{\mathrm{s}}}{\partial Y^2} + K_{\mathrm{r}}H(\theta_{\mathrm{f}} - \theta_{\mathrm{s}}) \tag{11}$$

where the parameters arises in the non-dimensionalization are defined as

$$\lambda = \frac{\alpha_{\mathrm{f}}}{\alpha_{\mathrm{s}}}; \quad K_{\mathrm{r}} = \frac{\varphi k_{\mathrm{f}}}{(1-\varphi)k_{\mathrm{s}}}; \quad H = \frac{hL^2}{\varphi k_{\mathrm{f}}Ra} \tag{12}$$

The initial and boundary conditions Eq. (5) become:

$$U(X,Y,0) = U(X,Y,0) = 0, \quad \theta_{\mathrm{f}}(X,Y,0) = \theta_{\mathrm{s}}(X,Y,0) = 0 \tag{13a}$$

$$U(0,Y,\tau) = 0, \quad \theta_{\mathrm{f}}(0,Y,\tau) = \theta_{\mathrm{s}}(0,Y,\tau) = 0 \tag{13b}$$

$$V(X,0,\tau)=0,\quad \theta_{\mathrm{f}}(X,0,\tau)=\theta_{\mathrm{s}}(X,0,\tau)=1+\varepsilon\sin(\Omega\tau) \tag{13c}$$

$$U(X,\infty,\tau)=0,\quad \theta_{\mathrm{f}}(X,\infty,\tau)=\theta_{\mathrm{s}}(X,\infty,\tau)=0 \tag{13d}$$

It is of practical importance to determine the average total heat transfer (by fluid and solid) per unit area from the vertical plate, which can be calculated as

$$\bar{q}_{\mathrm{t}}=\frac{1}{L}\int_0^L q_{\mathrm{t}}(x)\,\mathrm{d}x=\frac{-1}{L}\int_0^L\left\{\varphi k_{\mathrm{f}}\left(\frac{\partial T_{\mathrm{f}}}{\partial y}\right)_{y=0}+(1-\varphi)k_{\mathrm{s}}\left(\frac{\partial T_{\mathrm{s}}}{\partial y}\right)_{y=0}\right\}\mathrm{d}x \tag{14}$$

From which it can be shown that, the time dependent total average Nusselt number is

$$\frac{\overline{Nu_{\mathrm{t}}}(\tau)}{\sqrt{Ra(\tau)}}=\frac{-1}{[\theta_{\mathrm{w}}(\tau)]^{3/2}(K_{\mathrm{r}}+1)}\times\int_0^1\left\{K_{\mathrm{r}}\left(\frac{\partial\theta_{\mathrm{f}}}{\partial Y}\right)_{Y=0}+\left(\frac{\partial\theta_{\mathrm{s}}}{\partial Y}\right)_{Y=0}\right\}\mathrm{d}X \tag{15}$$

where $\overline{Nu_{\mathrm{t}}}(\tau)=\bar{h}(\tau)L/k_{\mathrm{eff}}$ is the time dependent average Nusselt number based on the time dependent average heat transfer coefficient and the effective thermal conductivity ($k_{\mathrm{eff}}=\varphi k_{\mathrm{f}}+(1-\varphi)k_{\mathrm{s}}$), and $Ra(\tau)=g\beta\{T_{\mathrm{w}}(t)-T_{\infty}\}LK/(\varphi\nu\alpha_{\mathrm{f}})$ is the time dependent Rayleigh number. It is assumed that $\theta_{\mathrm{w}}(\tau)\neq 0$ where the present problem arises. It can be shown that the time dependent average Nusselt number for the fluid and solid phases are

$$\frac{\overline{Nu_{\mathrm{f}}}(\tau)}{\sqrt{Ra(\tau)}}=\frac{-1}{[\theta_{\mathrm{w}}(\tau)]^{3/2}}\int_0^1\left(\frac{\partial\theta_{\mathrm{f}}}{\partial Y}\right)_{Y=0}\mathrm{d}X \tag{16a}$$

$$\frac{\overline{Nu_{\mathrm{s}}}(\tau)}{\sqrt{Ra(\tau)}}=\frac{-1}{[\theta_{\mathrm{w}}(\tau)]^{3/2}}\int_0^1\left(\frac{\partial\theta_{\mathrm{s}}}{\partial Y}\right)_{Y=0}\mathrm{d}X \tag{16b}$$

It is can be seen from Eqs. (10), (15) and (16) that, when $H\to 0$ and $K_{\mathrm{r}}\to\infty$; leads to the equilibrium model. The thermal equilibrium model solution can be recovered also when the heat transfer coefficient parameter $H\to\infty$ which leads to same temperatures of the solid and fluid in the boundary layer.

3. Numerical scheme

The energy equations (10) and (11) are integrated over a control volume using the fully implicit scheme which is unconditionally stable. The power-law scheme is used for the convection-diffusion formulation [30]. Finally, the finite-difference equation corresponding to the continuity Eq. (8) is developed using the expansion point $\left(i+1,j-\frac{1}{2}\right)$, where i and j are the indices along X and Y respectively [31]. The resulting finite-difference equation is

$$V_{i+1,j}=V_{i+1,j-1}-\frac{\Delta Y_{\mathrm{w}}}{2(\Delta X_n)}\left(U_{i+1,j}+U_{i+1,j-1}-U_{i,j}-U_{i,j-1}\right) \tag{17}$$

where ΔY_{w} and ΔX_n are the grid spaces west and north of the (i,j) point respectively. The solution domain, therefore, consists of grid points at which the discretization equations are applied. In this domain X by definition varies from 0 to 1. But the choice of the value of Y, corresponding to $Y=\infty$, has an important influence on the solution. In the present study the value of Y corresponding to $Y=\infty$ is taken $Y=14$ following Jang and Ni [8]. Further larger values of Y produced the results with indistinguishable difference. The stretched grid has been selected in both X and Y direction such that the grid points clustered near the wall and near the leading edge of the flat plate as there are steep variation of the velocities and temperatures in these regions.

The algorithm needs iteration for the coupled Eqs. (8)–(11). The convergence condition used for the dependent variables θ_{f}, θ_{s}, U, and V is

$$\mathrm{Max}\left|\frac{\Phi^n-\Phi^{n-1}}{\Phi^n}\right|<10^{-5} \tag{18}$$

where Φ is the general dependent variable and the superscript n represents the iteration step number. The time increment $\Delta\tau=0.001$ is used for the isothermal wall case, and $\Delta\tau=2\pi/(1000\Omega)$ is used for the oscillation surface temperature case and even smaller in some case of small values of the non-dimensional frequency.

The steady-state temperature profile of the fluid and solid phases are shown in Fig. 1 for the $H=0$ and $K_{\mathrm{r}}=1000$ and the case $H=1000$ and $K_{\mathrm{r}}=1000$ which represent the equilibrium model for the isothermal plate ($\varepsilon=0$). The results of the equilibrium model found by Jang and Ni [8] are presented in Fig. 1 for comparison.

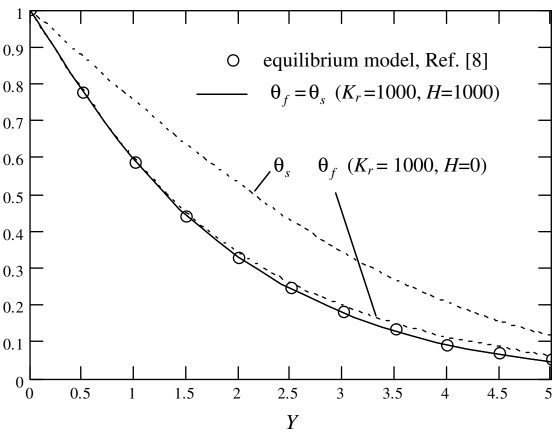

Fig. 1. Steady-state temperature profiles at $X=1$, $\varepsilon=0$.

Good agreement can be observed between the present results and the numerical results of Jang and Ni [8]. The steady-state value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)} = \overline{Nu}_f(\tau)/\sqrt{Ra(\tau)} = \overline{Nu}_s(\tau)/\sqrt{Ra(\tau)} = 0.888$ is found when $H = 1000$ and $K_r = 1000$ with $\lambda = 1.0$. While $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)} = 0.884$ is found when $H = 0$ and $K_r = 1000$ with $\lambda = 1.0$. The exact value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)} = 0.888$ is found according to the similarity solution of the equilibrium model in the free convection boundary layer flow along isothermal plate embedded in a porous medium [7]. Moreover the steady-state value of the local $Nu/\sqrt{Ra}$ is found as 0.446 at the upper end of the plate ($X = 1$) comparing with the value of 0.444 obtained from the similarity solution [7]. The difference between the present results and the similarity solution [7] for both the local and average value of the Nusselt number is less than 0.5%. The above results provided confidence to the accuracy of the numerical algorithm used for the present problem in comparison with the equilibrium model.

4. Results and discussion

The transient variation of $\overline{Nu}_f(\tau)/\sqrt{Ra(\tau)}$, $\overline{Nu}_s(\tau)/\sqrt{Ra(\tau)}$ and $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ for the isothermally suddenly heated plate ($\varepsilon = 0$) are shown in Fig. 2 for three different cases. ($H = 0$, $K_r = 1000$), ($H = 1000$, $K_r = 1000$) and ($H = K_r = 1$). The results show that, when $H = 0$ and $K_r = 1000$, the transient variation of the average Nusselt number for the fluid is exactly same as the total average Nusselt number with the steady-state value of 0.884 but the average Nusselt number for the solid is lower. The thermal equilibrium solution is obtained for high H and high value of K_r. The transient variation of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$, $\overline{Nu}_f(\tau)/\sqrt{Ra(\tau)}$ and $\overline{Nu}_s(\tau)/\sqrt{Ra(\tau)}$ are forming the continues upper curve in Fig. 2 when ($H = 1000$, $K_r = 1000$). While for the case when $H = 1$ and $K_r = 1$, the values of average Nusselt number for the fluid and the total average Nusselt number are reduced and the values of average Nusselt number for the solid are increased. The total average Nusselt number values are exactly in between the values of the average Nusselt number for the fluid and for the solid according to Eqs. (15) and (16).

The variation of the steady-state $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ with the thermal conductivity ratio parameter K_r in the range ($K_r = 0$–10) for different values of the heat transfer coefficient parameter H and fixed value of the thermal diffusivity ratio $\lambda = 1$ is shown in Fig. 3 for the isothermal plate ($\varepsilon = 0$). It is observed that the steady-state value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ increases with the increase of the thermal conductivity ratio parameter K_r. It can be predicted from Fig. 3 that the steady-state of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ will approach 0.888 (equilibrium model solution) when K_r goes to infinity. Fig. 3 shows also that the thermal conductivity ratio parameter K_r has a significant effect on the average total Nusselt number, while the effect of the heat transfer coefficient parameter H is approximately insignificant in the case of the isothermal plate and there is no significant difference even for very small values of H (0 or 0.1). Therefore, the present study will focus on the effect of the sinusoidal temperature oscillation of the plate embedded in a porous medium for different value of the thermal conductivity ratio parameter K_r with fixed values of $H = 1$.

In this case the free convection process starts when the vertical plate temperature increases suddenly from the ambient temperature T_∞ to the average plate temperature $\overline{T}_w$. At this time the average Nusselt number goes to infinity. Then when the vertical plate temperature is oscillating, the temperature of the both phases at $Y = 0$ is oscillating and the total average Nusselt number will oscillates accordingly as a result of the oscillation of the solid and fluid average Nusselt number. It is found that the oscillation of the solid average Nusselt number is always with smaller values than that of the fluid phase. For very high values of the thermal conductivity ratio parameter ($K_r = 10^3$) the oscillation of the

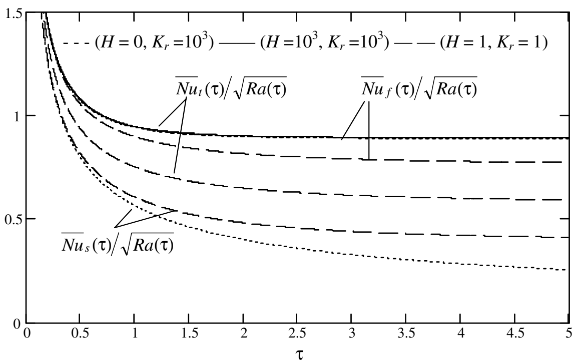

Fig. 2. Transient variation of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$, $\overline{Nu}_f(\tau)/\sqrt{Ra(\tau)}$ and $\overline{Nu}_s(\tau)/\sqrt{Ra(\tau)}$ for the thermally equilibrium and non-equilibrium cases.

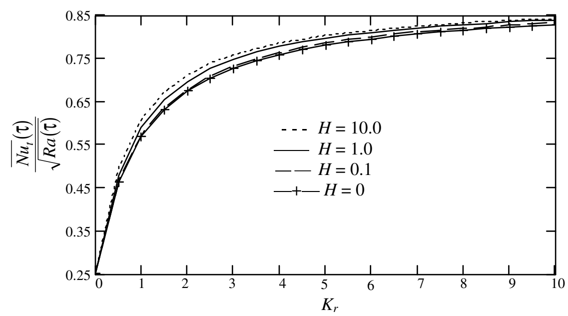

Fig. 3. Variation of the steady-state $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ with the thermal conductivity ratio parameter K_r for different values of H with $\lambda = 1$ and $\varepsilon = 0$.

total average Nusselt number will be same as that for the fluid phase, while for very small values of the thermal conductivity ratio parameter ($K_r = 10^{-3}$) the oscillation of the total average Nusselt number will be same as that for the solid phase. Fig. 4 shows the oscillation of the plate temperature with an amplitude of $\varepsilon = 0.1$ and frequency of $\Omega = 1$ and the effect of the plate temperature oscillation on the average Nusselt number. It can be observed that the oscillation of the average Nusselt numbers is slightly out of phase with the plate temperature oscillation for this case. It can be predicted that the phase difference between the oscillation of the average Nusselt numbers and the plate temperature oscillation is higher for higher frequency.

The oscillation of the total average Nusselt number becomes periodic oscillation after some periods. The last two oscillating periods in Fig. 4 are almost similar, which means that the oscillation of the total average Nusselt number becomes periodic oscillation.

The steady periodic oscillation of the total average Nusselt number is achieved when the amplitude and the cyclic averaged value of the total average Nusselt number become constant for different periods. Therefore, the following condition is considered for the steady periodic oscillation of the total average Nusselt number:

$$\frac{A\left(\overline{Nu_t}\right)^p - A\left(\overline{Nu_t}\right)^{p-1}}{A\left(\overline{Nu_t}\right)^p} \leqslant 10^{-3} \quad \text{and} \quad \frac{\overline{\overline{Nu_t}}^p - \overline{\overline{Nu_t}}^{p-1}}{\overline{\overline{Nu_t}}^p} \leqslant 10^{-3} \tag{19}$$

where the superscript p is the period number, and

$$A\left(\overline{Nu_t}\right) = \frac{1}{2}\left[\mathrm{Max}\left(\frac{\overline{Nu_t}(\tau)}{\sqrt{Ra(\tau)}}\right) - \mathrm{Min}\left(\frac{\overline{Nu_t}(\tau)}{\sqrt{Ra(\tau)}}\right)\right] \tag{20a}$$

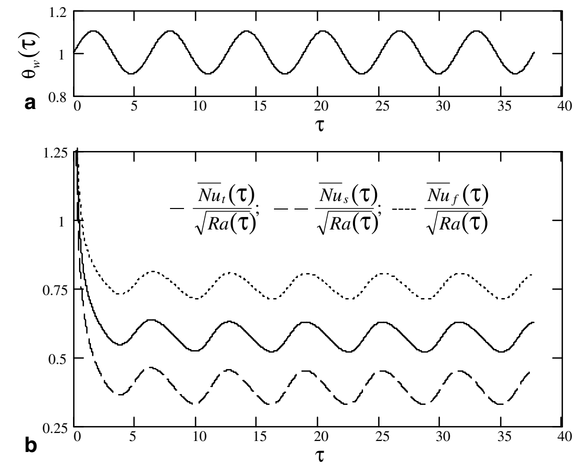

Fig. 4. Oscillation of (a) plate temperature (b) average Nusselt number. $\varepsilon = 0.1$; $\Omega = 1$; with $K_r = 1$, $H = 1$ and $\lambda = 1$.

for $\tau_0 \leqslant \tau \leqslant [\tau_0 + (2\pi/\Omega)]$ and

$$\overline{\overline{Nu_t}} = \frac{1}{(2\pi/\Omega)} \int_{\tau_0}^{\tau_0+(2\pi/\Omega)} \left(\frac{\overline{Nu_t}(\tau)}{\sqrt{Ra(\tau)}}\right) \mathrm{d}\tau \tag{20b}$$

Fig. 5 shows the variation of the temporal cyclic averaged values of $\overline{Nu_t}(\tau)/\sqrt{Ra(\tau)}$ defined in Eq. (20b) and designed as $\overline{\overline{Nu_t}}$ with the non-dimensional frequency Ω, at different amplitudes of the surface temperature oscillation (ε). Two different cases are presented in Fig. 5 which are same as those in Fig. 2 for the non-oscillation plate temperature, namely ($K_r = 1000$; $H = 0$) and ($K_r = 1$; $H = 1$). For both cases the value of $\overline{\overline{Nu_t}}$ is decreasing with the increase of either the non-dimensional frequency or the amplitude of the plate temperature oscillation.

The effect of the thermal conductivity ratio parameter (K_r) on $\overline{\overline{Nu_t}}$ is shown in Fig. 6 when the plate temperature oscillates at different amplitudes and constant non-dimensional frequency of $\Omega = 5$ and $\lambda = 1$. Once again for all the values of K_r the values of $\overline{\overline{Nu_t}}$ are decreasing with the increase of the amplitude of the plate temperature oscillation. Increasing the thermal conductivity ratio parameter (K_r) will enhance the free convective

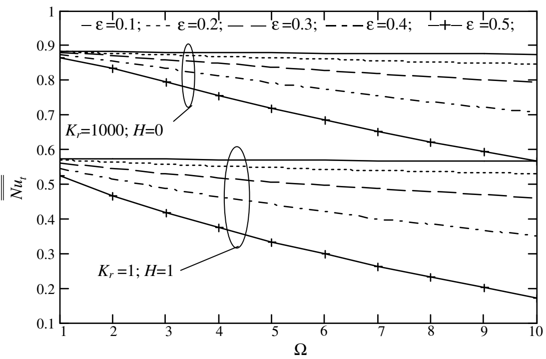

Fig. 5. Variation of $\overline{\overline{Nu_t}}$ with the non-dimensional frequency at the periodic state. $\lambda = 1$.

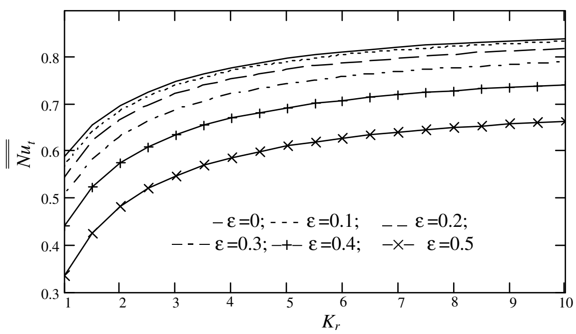

Fig. 6. Variation of $\overline{\overline{Nu_t}}$ at the periodic state with the thermal conductivity ratio parameter. $H = 1$, $\lambda = 1$ and $\Omega = 5$.

heat transfer for the non-oscillating and oscillating plate temperature and increase the values of $\overline{\overline{Nu_t}}$ as expected and shown in Fig. 6.

All the above results are presented for the case of equal thermal diffusivity for the fluid phase and the solid phase i.e., $\lambda = 1$. In the practice this ratio can be more or less than unity and it is important to consider the effect of this parameter for the present unsteady free convection problem. Amiri and Vafai [14] have considered values of $\lambda < 1$ for the transient analysis of incompressible flow through a packed bed. In the present work the effect of λ is studied for values less than and more than unity as shown in Fig. 7. The variation of $\overline{\overline{Nu_t}}$ with the thermal conductivity ratio parameter (K_r) when $\lambda = 1$ is chosen to be as a reference for the case when the plate temperature oscillates with $\varepsilon = 0.3$ and $\Omega = 5$. For all the values of K_r presented in Fig. 7, the values of $\overline{\overline{Nu_t}}$ increase with the decrease of the thermal diffusivity parameter (λ).

It is observed that when the plate temperature oscillates at high amplitude and high frequency the values of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ becomes negative for some instances of the oscillation period. Negative values of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ are observed for the cases of high values of the thermal diffusivity ratio (λ) and small values of the thermal conductivity ratio parameter (K_r). The oscillation of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ with time for the last period when the conditions (19) are satisfied is shown in Fig. 8 for different values of the amplitude of the plate temperature oscillation. It can be seen from Fig. 8 that for non-dimensional frequency of 5, when the amplitude of the plate temperature oscillation $\varepsilon \geqslant 0.3$, negative values are obtained for the particular parameter values of $K_r = 1$, $H = 1$, and $\lambda = 1$. As discussed above smaller values of K_r gives smaller values of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ and higher values of λ or Ω results in smaller values of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ also.

Negative values of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ means that there is no enough time to transfer the heat from the plate to the ambient medium and there will be some point in the boundary layer with fluid and/or solid temperature higher

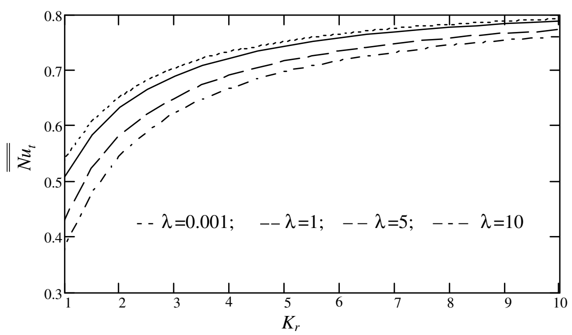

Fig. 7. Effect of the thermal diffusivity ratio λ on the variation of $\overline{\overline{Nu_t}}$ at the periodic state with the thermal conductivity ratio parameter. For $H = 1$, $\varepsilon = 0.3$ and $\Omega = 5$.

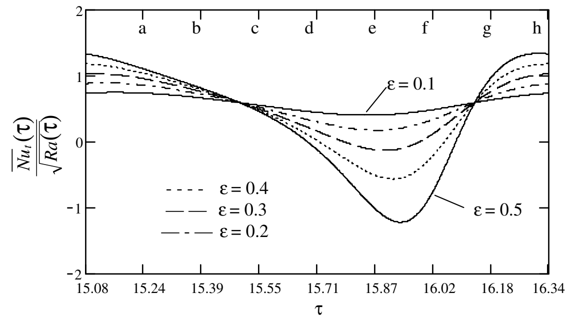

Fig. 8. Ultimate periodic oscillation of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ with τ at different values of ε with $\Omega = 5$, $K_r = 1$, $H = 1$, and $\lambda = 1$.

than the surface temperature from which heat will transfer partly to the wall, or some of the heat gained by the porous media in early stages will return back to it when its temperature drops.

A detailed investigation has been carried out to demonstrate this explanation when negative heat transfer happened. The period of the last cycle of the oscillation of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ is divided in to eight time steps (a)–(h) as shown in Fig. 8. At each time step the temperature profiles at the upper end of the plate ($X = 1$) are shown in Figs. 9 and 10 for fluid and solid phase respectively for $\varepsilon = 0.5$. It can be seen from Figs. 9 and 10 that the surface temperature is equal at the points (a, c), (d, h) and (e, g) but the temperature gradients are these points different which gives different values of the Nusselt number. The maximum value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ occurs near point (h) when the surface temperature increases from its minimum value to its average value as shown in Fig. 8. At point (h) the temperature profiles for both fluid and solid phases show maximum negative

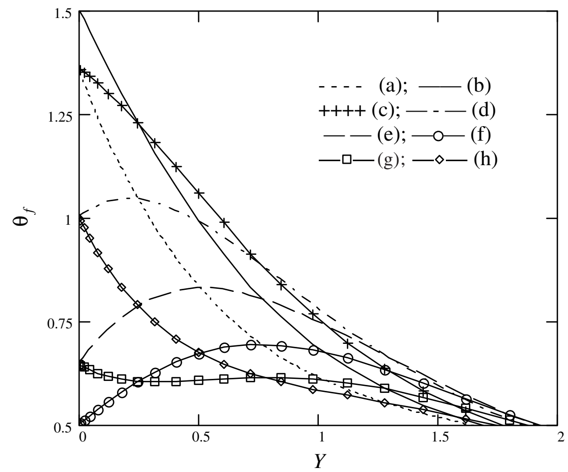

Fig. 9. Periodic state fluid temperature profiles at different time steps (a–h in Fig. 8) for last cycle, $\varepsilon = 0.5$ and $\Omega = 5$ for $K_r = 1$, $H = 1$, and $\lambda = 1$.

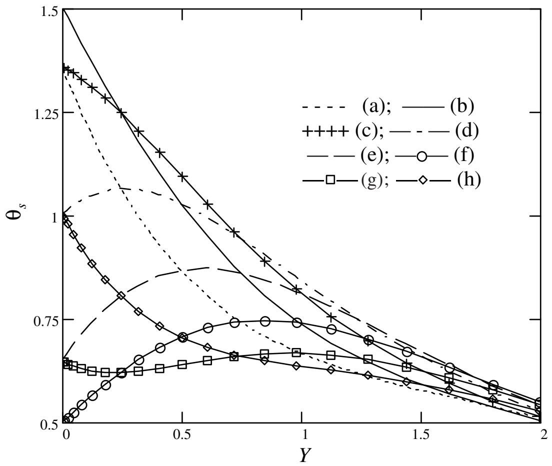

Fig. 10. Periodic state solid temperature profiles at different time steps (a–h in Fig. 8) for last cycle, $\varepsilon = 0.5$ and $\Omega = 5$ for $K_r = 1$, $H = 1$, and $\lambda = 1$.

temperature gradient at the plate ($Y = 0$) which leads to the maximum value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$. The minimum value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$ is negative and it occurs between points (e) and (f) when the surface temperature goes to its minimum value. The reason is that at points (e) and (f) the temperature profiles for both fluid and solid phases show maximum temperature gradient but in this case positive at $Y = 0$ which leads to the negative value of $\overline{Nu}_t(\tau)/\sqrt{Ra(\tau)}$.

5. Conclusions

In this paper, the effect of the periodic oscillation of the surface temperature on the periodic free convection from a vertical heated plate in a porous medium is investigated numerically. The non-equilibrium model is used in the present investigation, i.e., porous solid matrix and saturated fluid are not necessary to be at same temperature locally. The non-dimensional form of the two-dimensional transient laminar boundary layer equations are solved numerically using the fully implicit scheme. The results are effect by three parameters: (1) H, heat transfer coefficient parameter, (2) K_r, thermal conductivity ratio parameter, and (3) λ, thermal diffusivity ratio. Two additional parameters arise from the plate temperature oscillation which are the non-dimensional amplitude (ε) and frequency (Ω) of the plate temperature oscillation. The numerical results are presented for $0 \leqslant H \leqslant 10$, $0 \leqslant K_r \leqslant 10$, $0.001 \leqslant \lambda \leqslant 10$ with the plate temperature oscillation parameters $0 \leqslant \Omega \leqslant 10$ and $0 \leqslant \varepsilon \leqslant 0.5$. The results show that the effect of the thermal conductivity ratio parameter is more important than the heat transfer coefficient parameter and the thermal diffusivity ratio. Increasing the thermal conductivity ratio parameter leads to increasing the average Nusselt number. It is found also that increasing the amplitude and the frequency of the oscillating surface temperature will decrease the free convection heat transfer from the plate for any values of the other parameters. It is observed that when the plate temperature oscillates at high amplitude and high frequency the values of the average Nusselt number becomes negative for some instances of the oscillation period. Negative values of the average Nusselt number are observed for the cases of high values of the thermal diffusivity ratio (λ) and small values of the thermal conductivity ratio parameter (K_r). Negative values of the average Nusselt number means that there will be some point in the boundary layer with fluid and/or solid temperature higher than the surface temperature from which heat will transfer partly to the wall, or some of the heat gained by the porous media in early stages will return back to it when its temperature drops.

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